Section 8.3: The Integral Test and p-Series

A. Convergence and Divergence Tests (Cont.)

4. Theorem 8.10 - Integral Test: If \( f \) is positive, continuous, and decreasing for \( x \geq 1 \) and \( a_n = f(n) \), then

\[
\sum_{n=1}^{\infty} a_n \quad \text{and} \quad \int_{1}^{\infty} f(x) \, dx
\]

either both converge or both diverge.

Examples: 2, 6, 8

5. Theorem 8.11 - p-Series Test: The p-series

\[
\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \ldots
\]

- If \( p > 1 \), then the series converges
- If \( p \leq 1 \), then the series diverges

B. Additional Ideas

- When \( p = 1 \): \( \sum_{n=1}^{\infty} \frac{1}{n} \) is called the harmonic series.

- \( \lim_{n \to \infty} \frac{1}{n} = 0 \), converges as a sequence, but \( \sum_{n=1}^{\infty} \frac{1}{n} \to \infty \) and diverges as a series.

Examples: 14, 18

C. If \( \sum_{n=1}^{\infty} a_n \) converges to \( S \), then the remainder \( R_N = S - S_N \) is bounded by

\[
0 \leq R_N \leq \int_{N}^{\infty} f(x) \, dx.
\]

Examples: 38, (46)
Other Examples: 54, 56, 60, 62