Section 8.2: Series and Convergence

A. \( \sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \ldots + a_n + \ldots \) is an infinite series.

B. Definition of Convergent and Divergent Series
For the infinite series \( \sum a_n \), the nth partial sum is given by
\[
S_n = \sum_{n=1}^{N} a_n = a_1 + a_2 + a_3 + \ldots + a_n
\]
If the sequence of partial sums converges to \( S \), then the series converges. The limit \( S \) is called the sum of the series.
\[
S = a_1 + a_2 + a_3 + \ldots + a_n + \ldots
\]
If \( \{S_n\} \) diverges, then the series diverges.
Examples: 4, 6

C. Convergence and Divergence Tests
1. Theorem 8.6: Geometric Series Test, \( \sum_{n=0}^{\infty} ar^n \) a geometric series.
   - If \(|r| < 1\), the series converges to \( \frac{a}{1 - r} \)
   - If \(|r| \geq 1\), the series diverges

Example: 10, 24, 30
2. Theorem 8.9: Divergent Test: Consider the series, \( \sum_{n=1}^{\infty} a_n \)

- If \( \lim_{n \to \infty} a_n \neq 0 \), then the series diverges
- If \( \lim_{n \to \infty} a_n = \text{DNE} \), then the series diverges
- If \( \lim_{n \to \infty} a_n = 0 \), no conclusion can be made. However if the series does converge, then \( \lim_{n \to \infty} a_n = 0 \).

Examples: 14, 16

3. Telescoping Series are of the form \( (b_1 - b_2) + (b_2 - b_3) + (b_3 - b_4) + \ldots \)

- The series will only converge if and only if \( b_n \) approaches a finite number as \( n \) approaches infinity.

Examples: 36, 54, 56, 74

D. Writing repeating decimals as a geometric series and find the infinite sum.