Section 7.7: Indeterminate Forms and L’Hopital’s Rule

A. Theorem 7.3: The Extended Mean Value Theorem
If f and g are differentiable on an open interval (a,b) and continuous on [a,b] such that g’(x) ≠ 0 for any x in (a,b), then there exists a point c on (a,b) such that

\[
\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}
\]

B. Theorem 7.4: L’Hopital’s Rule

Let f and g be functions that are differentiable on an open interval (a,b) containing c, except possibly at c itself. Assume g’(x) ≠ 0 for all x in (a,b), except possibly at c itself. If the limit of f(x)/g(x) as x approaches c produces an indeterminant form (0/0, ±∞/±∞) then

\[
\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}
\]

- L’Hopital’s Rule can be applied more than once.
- If you have other indeterminate forms such as
  0 \cdot \infty, 1^\infty, 0^0 convert to an indeterminant form that applies to L’Hopital’s Rule.
    - Using “ln” can be helpful in evaluating the limit.
    - If you have two fractions try combining them.

Examples: 6, 8ab, 18, 22
C. Summary

1. Indeterminant Forms: 0/0, \( \infty/\infty \), \( \infty - \infty \), 0*\( \infty \), 0\(^0\), 1\(^\infty\), and \( \infty^0 \)

2. Determinant Forms:

\[ \infty + \infty \rightarrow \infty \]

\[ -\infty - \infty \rightarrow -\infty \]

\[ 0^\infty \rightarrow 0 \]

\[ 0^{-\infty} \rightarrow \infty \]

3. Who wins the race to infinity?

\[ g(x) = e^{nx}, \quad n>0 \]

\[ f(x) = x^m, \quad m>0 \]

\[ h(x) = (\ln x)^n, \quad n>0 \]

Examples: 28, 32, 40, 46, 52