Section 6.4: Arc Length and Surfaces of Revolution

A. Definition of Arc Length

Let the function given by $y = f(x)$ represent a smooth curve on the interval $[a,b]$. The arc length of $f$ between $a$ and $b$ is

$$ s = \int_{a}^{b} \sqrt{1 + [f'(x)]^2} \, dx $$

Similarly, for a smooth curve given by $x = g(y)$, the arc length of $g$ between $c$ and $d$ is

$$ s = \int_{c}^{d} \sqrt{1 + [g'(y)]^2} \, dy $$

Examples: 4, 16

B. Definition of a Surface of Revolution: If a graph of a continuous function is revolved about a line, the resulting surface is a surface of revolution.

C. Definition of Area of a Surface Area: Let $y = f(x)$ have a continuous derivative on the interval $[a,b]$. The area $S$ of the surface of revolution formed by revolving the graph of $f$ about a horizontal or vertical axis is

$$ S = 2\pi \int_{a}^{b} r(x) \sqrt{1 + [f'(x)]^2} \, dx $$

where $r(x)$ is the distance between the graph of $f$ and the axis of revolution. If $x = g(y)$ on the interval $[c,d]$, then the surface area is

$$ S = 2\pi \int_{c}^{d} r(y) \sqrt{1 + [g'(y)]^2} \, dy $$

Examples: 34, 38