Section 6.1: Area of a Region Between Two Curves

A. Area of a Region Between Two Curves

If \( f \) and \( g \) are continuous on \([a,b]\) and \( g(x) \leq f(x) \) for all \( x \) in \([a,b]\), then the area of the region bounded by the graphs of \( f \) and \( g \) and the vertical lines \( x = a \) and \( x = b \) is

\[
\int_{a}^{b} [f(x) - g(x)] \, dx
\]

To find the area bounded by two intersecting graphs, you will need to calculate the points of intersection by setting the two functions equal to each other. The upper and lower limits of integration will determined by the points of intersection. Looking at a graph on your calculator is very helpful. Sometimes there may be more than two points of intersection.

B. Integration as an Accumulation Process

\[ A = (\text{height})(\text{width}) \rightarrow \Delta A = [f(x) - g(x)] \Delta x \rightarrow A = \int_{a}^{b} [f(x) - g(x)] \, dx \]

See Illustration on Page 417

Examples: 2, 6, 20, 24, 42, 54