Section 5.1: Differential Equations: Growth and Decay

A. Solving Differential Equations using Separation of Variables.
   - Rewrite the equation so that each variable occurs on only one side of the equation
   - Using Leibniz notation (dy/dx) is often preferred.

B. In many applications, the rate of change of a variable is proportional to the value of y. If y is a function of time t, the proportion can be written as follows.
   \[
   \frac{dy}{dt} = ky
   \]
   - If the rate of change is inversely proportional:
   \[
   \frac{dy}{dt} = \frac{y}{k}
   \]
   The solution of this differential equation is given by

Theorem 5.1: If y is a differentiable function of t such that y > 0 and y’ = ky, for some constant k, then
   \[
   y = Ce^{kt}
   \]
   C is the initial value of y, and k is the proportionality constant. Exponential growth occurs when k > 0, and exponential decay occurs when k < 0

C. Continuous Interest: \(A = Pe^{rt}\)

D. Newton’s Law of Cooling (optional) – The rate of change in the temperature of an object is proportional to the difference between the object’s temperature and the temperature of the surrounding medium. See page 378

Examples: 2, 4, 12, 16, 28, 37, 44