System of Equations

A system of equations is more than one equation.

Examples

\begin{align*}
\begin{cases}
y = 3x + 5 & \quad 2x - 3y = 12 & \quad 5x + 2y - 4z = 8 \\
y = -4x + 2 & \quad 5x + 2y = 20 & \quad 2x + 3y + 5z = 10 \\
\end{cases}
\end{align*}

We indicate the size of the system by the number of equations to the number of variables. $m \times n$ system

Examples

\begin{align*}
\begin{cases}
y = 5x + 4 & \quad 2x - 3y = 6 & \quad 3x - 2y + 5z = 6 \\
y = -3x + 1 & \quad 5x + 3y - 4z = 8 & \quad 2x + 3y - 4z = 8 \\
\end{cases}
\end{align*}

Solving 2x2 systems graphically

1. Write the equations in standard form.
2. Put equations in calculator and set the window to show intersection.
3. Zoom in to the intersection.

Note: The intersection is where the curves (lines) cross.

Example

\begin{align*}
y &= 3x - 5 \\
4x + 2y &= 10
\end{align*}

Example

\begin{align*}
3x + 4y &= -6 \\
2x - 3y &= 13
\end{align*}
Example
\[5x - 4y = 28\]
\[-4x + 7y = 8\]

Homework:
Graph on calculator and find the intersection.

1. \[y = 3x - 5\]
\[y = -2x + 5\]

2. \[y = 5x - 2\]
\[y = -3x + 14\]

3. \[3x + 2y = 1\]
\[x + 3y = 5\]

4. \[5x - 2y = 4\]
\[3x + 2y = 12\]

5. \[2x + 3y = 6\]
\[4x + 2y = -4\]

6. \[7x + 8y = 16\]
\[-5x + 2y = 6\]

7. \[3x + 2y = 8\]
\[-5x + 4y = 20\]

8. \[6x + 3y = 12\]
\[y = 5x - 8\]

9. \[12x - 3y = 18\]
\[5x + 10y = 20\]

10. \[7x + 3y = 15\]
\[8x - 5y = 20\]

11. \[y = .04x - 5\]
\[y = -.07x + 3\]

12. \[5x - 7y = 12\]
\[4x + 3y = 10\]

13. \[2x - 4y = 12\]
\[5x + 3y = 9\]

14. \[3x - y = 15\]
\[2x + y = 5\]

15. \[5x - 4y = 10\]
\[3x - 2y = 4\]
Substitution Method

Solving systems of equations using substitution.
1. Solve for a variable in one of the equations.
2. Substitute the expression from step 1 into other equation.
3. Solve for the variable left.
4. Substitute the value found in step 3 into equation used in step 1 and solve.

Example

\[3x + y = 7\]
\[5x - 2y = 8\]

Example

\[3x + 2y = 7\]
\[x - 3y = 6\]

Example

\[4x + 3y = 13\]
\[5x + 2y = 4\]
Example
\[3x - 4y = 18\]
\[5x + 2y = 11\]

**Homework:**
**Substitution Method**

1. \[x + y = 5\]
   \[2x - y = 4\]

2. \[x - 2y = 7\]
   \[3x - y = 6\]

3. \[2x + y = 7\]
   \[x + 3y = 6\]

4. \[4x + 2y = 12\]
   \[x - 5y = 25\]

5. \[x - 3y = 17\]
   \[3x - y = 11\]

6. \[4x + y = 18\]
   \[3x - 2y = 8\]

7. \[2x + 3y = -9\]
   \[5x + y = 10\]

8. \[x + 2y = 1\]
   \[2x + 4y = 2\]

9. \[3x - 4y = 8\]
   \[x - y = 5\]

10. \[x + y = 13\]
    \[2x - 3y = 1\]

11. \[x + 2y = 7\]
    \[5x - 3y = 9\]

12. \[3x + y = 8\]
    \[4x + 3y = 9\]
Elimination Method

What is the opposite of 3?

What does the sum (result of adding) of 3 and its opposite give?

What is the opposite of -5?

What does the sum (result of adding) of -5 and its opposite give?

Solving system of equations by elimination of variables.

1. Multiply each equation by a value such that the coefficients of one variable are the same or opposites.
2. If the coefficients are the same then subtract the equations.
   If the coefficients are opposites then add the equations.
3. Solve for the variable left.
4. Repeat steps 1-3 on the other variable.

Note: Step 4 could use substituting the value from step 3 into one of the original equations and solving for other variable.

Example

\[3x + 2y = 16\]
\[4x - 3y = -7\]

Eliminating \(y\)
\[3 \cdot (3x + 2y = 16) \Rightarrow 9x + 6y = 48\]
\[2 \cdot (4x - 3y = -7) \Rightarrow 8x - 6y = -14\]

\[17x = 34\]
\[x = 2\]

Eliminating \(x\)
\[4 \cdot (3x + 2y = 16) \Rightarrow 12x + 8y = 64\]
\[3 \cdot (4x - 3y = -7) \Rightarrow 12x - 9y = -21\]

\[17y = 85\]
\[y = 5\]

Example

\[5x - 2y = 23\]
\[7x + 3y = 9\]
Example
\[2x + 3y = 8\]
\[4x - 5y = 10\]

Homework:
**Elimination Method**

1. \[3x + 2y = 8\]
   \[4x - y = 7\]
2. \[5x + 2y = 4\]
   \[3x - 4y = 18\]
3. \[-x + 3y = -5\]
4. \[3x + 4y = 14\]
   \[7x - 5y = 4\]
5. \[4x - 2y = 12\]
   \[-3x + 5y = 5\]
6. \[7x + 2y = 25\]
   \[4x - 3y = 6\]
7. \[3x - 5y = 17\]
   \[6x - 2y = 2\]
8. \[3x - 5y = 8\]
   \[2x + 2y = 9\]
9. \[8x - 3y = 18\]
10. \[2x + 7y = 12\]
    \[3x - 4y = 8\]
11. \[4x - 2y = 12\]
    \[5x + 3y = 4\]
12. \[5x + 7y = 19\]
    \[3x + 2y = 18\]
13. \[2x - 5y = 1\]
    \[3x - 7y = 1\]
14. \[5x + 2y = 8\]
    \[x + 3y = 10\]
15. \[4x + y = 9\]
    \[3x - 2y = 11\]
16. \[4x + 3y = 3\]
    \[2x - 2y = -16\]
17. \[3x - y = 14\]
    \[5x + 4y = 12\]
18. \[5x + 3y = 9\]
    \[4x + 2y = 10\]