13.3: Changes of Variables: Polar Coordinates

1. **Some integrals are much easier to evaluate in polar form than in rectangular form. Recall from section 9.4:**

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$r^2 = x^2 + y^2$$
$$\tan\theta = \frac{y}{x}$$

2. **Theorem 13.3: Change of Variables to Polar Form**

   Let R be a plane region consisting of all points (x,y) = (r cosθ, r sinθ) satisfying the conditions $0 \le g_1(\theta) \le r \le g_2(\theta),\ \ \alpha \le \theta \le \beta$, where $0 \le (\beta - \alpha) \le 2\pi$. If g_1 and g_2 are continuous on [α,β] and f is continuous on R, then

$$\int_R\int f(x,y)dA = \int_{\alpha}^{\beta}\int_{g_1(\theta)}^{g_2(\theta)} f(r\cos\theta, r\sin\theta)rdrd\theta$$

   The region R is restricted by two basic types: **r-simple** regions and **θ-simple** regions.

   Be sure to notice the extra factor of r in the integrand. This comes from the formula for are of a polar section. In differential notation, $dA = rdrd\theta$, which indicates that area of a polar sector increases as you move away from the origin.

3. **Just as with rectangular coordinates, the double integral $\int_R\int dA$ can be used to find the area of a region in the plane.**