13.2: Double Integrals and Volume

1. **Definition of Double Integral**

   If f is defined on a closed, bounded region R in the xy-plane, then the double integral of f over R is given by

$$\int_R \int f(x,y)dA = \lim_{\|\Delta\| \to o} \sum_{i=1}^{n} f(x_i, y_i)\Delta A_i$$

2. **Volume of a Solid Region**

   If f is integrable over a plane region R and f(x,y) ≥ 0 for all (x,y) in R, then the volume of the solid region that lies above R and below the graph of f is defined as

$$V = \int_R \int f(x,y)dA$$

3. **Theorem 13.1: Properties of Double** Integrals (See page 946)

   Double integrals share many properties of single integrals

4. **Normally, the first step in evaluating a double integral is to rewrite it as an iterated integral.**

5. **Theorem 13.2: Fubini's Theorem**

   Let f be continuous on a plan region R.

   - If R is defined by $a \le x \le b$ and $g_1(x) \le y \le g_2(x)$, where g_1 and g_2 are continuous on [a,b], then

$$\int_R \int f(x,y)dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)dydx$$

   - If R is defined by $c \le x \le d$ and $h_1(y) \le x \le h_2(y)$, where h_1 and h_2 are continuous on [c,d], then

$$\int_R \int f(x,y)dA = \int_c^d \int_{h_1(x)}^{h_2(x)} f(x,y)dxdy$$