13.1: Iterated Integrals and Area of the Plane

1. **We can integrate functions of several variables is a similar way we differentiated functions of several variables in Chapter 12.**

$$\int_{h_1(y)}^{h_2(y)} f_x(x,y)dx = f(x,y)\Big]_{h_1(y)}^{h_2(y)} = f(h_2(y),y) - f(h_1(y),y) \quad \text{with respect to x}$$

$$\int_{g_1(y)}^{g_2(y)} f_y(x,y)dy = f(x,y)\Big]_{g_1(y)}^{g_2(y)} = f(x,g_2(x)) - f(x,g_1(x)) \quad \text{with respect to y}$$

2. **Iterated integrals are usually written as**

$$\int_a^b \int_{g_1(x)}^{g_2(x)} f_x(x,y)dydx \quad \text{and} \quad \int_c^d \int_{h_1(y)}^{h_2(y)} f_x(x,y)dxdy$$

The **inside limits** of integration can be a variable with respect to the out variable of integration. However, the **outside limits** of integration must be constant with respect to both variables of integration. After performing the inside integration, you obtain a "standard" definite integral. Together the two intervals of integration determine **the region of integration R** of the iterated integral.

3. **Area of a Region in the Plane**

- If R is defined by $a \le x \le b$ and $g_1(x) \le y \le g_2(x)$, where g_1 and g_2 are continuous on [a,b], then the area of R is given by

$$A = \int_a^b \int_{g_1(x)}^{g_2(x)} dydx$$ *This type of region is called vertically simple*

- If R is defined by $c \le x \le d$ and $h_1(y) \le x \le h_2(y)$, where h_1 and h_2 are continuous on [c,d], then the area of R is given by

$$A = \int_c^d \int_{h_1(y)}^{h_2(y)} dxdy$$ *This type of region is called horizontally simple.*

4. **One order of integration will often produce an simpler integration problem. Try to always choose the order that is simpler.**

In this section, most of the attention is given to procedures. The next section will deal more with why iterated integrals are needed.