12.1: Introduction to Functions of Several Variable

1. **Definition of a Function of Two Variables**

   Let D be a set of ordered pairs of real numbers. If to each ordered pair (x,y) in A there corresponds a unique real number f(x,y), then f is called **a function of x and y**. The set D is the **domain** of f, and the corresponding set of values for f(x,y) is the **range** of f.

   Example: $f(x,y) = x^2 y^2 + x$

   For a function give by z = f(x,y), x and y are called the **independent variables** and z is called the **dependent variable**.

2. Functions of several variables can be combined in the same ways as functions of single variables.

$$(f \pm g)(x,y) = f(x,y) + g(x,y)$$
$$(fg)(x,y) = f(x,y)g(x,y)$$
$$\frac{f}{g}(x,y) = \frac{f(x,y)}{g(x,y)}$$
$$(g \circ h)(x,y) = g(h(x,y))$$

   A function that can be expressed as a sum of functions of the form $cx^m y^n$ is called a **polynomial function of two variables**. A **rational function** is the quotient of two polynomial functions.

3. The graph of a function f of two variables can be interpreted geometrically as a surface in space. To sketch a surface in space by hand, it helps to use traces in the planes parallel to the coordinate planes.

4. **Level Curves**

   A second way to visualize a function of two variables is to use a **scalar field** in which the scalar z = f(x,y) is assigned a point (x,y). A **scalar field** can be characterized by **level curves** (or **contour lines**) along which the value of f(x,y) is contstant. See examples on page 841.

5. **Level Surfaces**

   The concept of a level curve can be extended by one dimension to define a level surface. If f is a function of 3 variables and c is a constant, the graph of **f(x,y,z) = c is a level surface of the function f.**