12.10: Lagrange Multipliers

1. **The scalar λ is called a Lagrange Multiplier**

   Theorem 12.19: Lagrange's Theorem

   Let f and g have continuous first partial derivatives such that f has an extremum at a point (x_0,y_0) on the smooth constraint curve g(x,y) = c. If $\nabla g(x_0, y_0) \neq \mathbf{0}$, then there is a real number λ such that

$$\nabla f(x_0, y_0) = \lambda \nabla g(x_0, y_0)$$

2. **Method of Lagrange Multipliers**

   Let f and g satisfy the hypothesis of Lagrange's Theorem, and let f have a minimum subject constraint g(x,y) = c. To find the minimum or maximum of f, use the following steps,

   - Simultaneously solve the equations $\nabla f(x_0, y_0) = \lambda \nabla g(x_0, y_0)$ and g(x,y) = c by solving the following system of equations

$$f_x(x, y) = \lambda \nabla g_x(x, y)$$
$$f_y(x, y) = \lambda \nabla g_y(x, y)$$
$$g(x, y) = c$$

   - Evaluate f at each solution point obtained from above. The largest value yields the maximum of f subject to the constraint g(x,y) = c, and the smallest value yields the minimum of f.

3. **Economist call the Lagrange multiplier obtained in a production function the marginal productivity of money**, which tells for each addition dollar spent on production, λ additional units of product can be produced.

4. **For optimization problems involving two constraints g and h**, a second Lagrange Multiplier, μ, is introduced. You then need to solve the equation

$$\nabla f = \lambda \nabla g + \mu \nabla h$$

   Where the gradient vectors are not parallel.

5. The system of equations that arise in the Method of Lagrange Multipliers is not, in general, a linear system. The solution often requires ingenuity.