11.3: Velocity and Acceleration

A. **As an object moves along a curve in the plane**, the coordinates x and y of its center of mass are each functions of time t. Rather than using f and g to represent these two functions, it is convenient to write x = x(t) and y = y(t). So, the **position vector r**(t) takes the form

$$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$$

B. Definition of Velocity and Acceleration

If x and y are twice-differentiable functions of t, and **r** is a vector valued function given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, then the velocity vector, acceleration vector, and the speed at time t are as follows.

$$\textbf{Velocity} = \mathbf{v}(t) = \mathbf{r}'(t) = x'(t)\mathbf{i} + y'(t)\mathbf{j}$$

$$\textbf{Acceleration} = \mathbf{a}(t) = \mathbf{r}''(t) = x''(t)\mathbf{i} + y''(t)\mathbf{j}$$

$$\textbf{Speed} = \|\mathbf{v}(t)\| = \|\mathbf{r}'(t)\| = \sqrt{[x'(t)]^2 + [y'(t)]^2}$$

C. **For motion along a space curve, the definitions are similar**. That is if

$$\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$$

$$\textbf{Velocity} = \mathbf{v}(t) = \mathbf{r}'(t) = x'(t)\mathbf{i} + y'(t)j + z'(t)\mathbf{k}$$

$$\textbf{Acceleration} = \mathbf{a}(t) = \mathbf{r}''(t) = x''(t)\mathbf{i} + y''(t)\mathbf{j} + z''(t)\mathbf{k}$$

$$\textbf{Speed} = \|\mathbf{v}(t)\| = \|\mathbf{r}'(t)\| = \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}$$

D. Projectile Motion

For a projectile of mass m, the force due to gravity is F = -mg**j** where the gravitational constant g = 32 feet per second per second or 9.81 meters per second per second. By Newton's Second Law of Motion, this same force produces an acceleration a = a(t), and satisfies the equation F = m**a**. Consequently, the acceleration of the projectile is given by m**a** = -mg**j** , which implies **a** = -g**j**

The position vector can be written in the form: $\mathbf{r}(t) = -\frac{1}{2}gt^2\mathbf{j} + t\mathbf{v}_o + \mathbf{r}_o$

E. Theorem 11.3: Position Function for a Projectile

Neglecting air resistance, the path of a projectile launched from an initial height h with initial speed v_o, and the angle of elevation θ is described by the vector function

$$\mathbf{r}(t) = (v_o \cos\theta)t\mathbf{i} + \left[h + (v_o \sin\theta)t - \frac{1}{2}gt^2 \right]\mathbf{j}$$