11.1: Vector-Valued Functions

A. **A space curve** C is the set of all ordered triples (f(t), g(t), h(t)) together with their defining parametric equations: x = f(t) y = g(t) z = h(t) where f, g, and h are continuous functions of t on an interval I.

B. Definition of a Vector-Valued Function

A function in the form :

$$\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} \quad \text{(Plane)}$$

$$\mathbf{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k} \qquad \text{(Space)}$$

is a vector valued function, where the component functions f, g, and h are real-valued functions of the parameter t. Vector valued functions are sometimes denoted as

$$\mathbf{r}(t) = \langle f(t), g(t) \rangle$$
$$\mathbf{r}(t) = \langle f(t), g(t), h(t) \rangle$$

C. Be sure to see the distinction between vector-valued functions r and the real-valued functions f, g, and h. All are functions of the real variable t, but r(t) is a vector, whereas f(t), g(t), and h(t) are real numbers for each value of t.

If t represents time, you can use vector valued functions to represent motion along a curve. You can use vector-valued functions to trace the graph of a curve. The terminal point of the position vector **r**(t) coincides with the point (x,y) or (x,y,z) on the curve given by the parametric equations. The arrowhead on the curve indicates the curves orientation by pointing in the direction of increasing values of t.

The domain of a vector-valued function is considered to be the intersection of the domains of f, g, and h.

D. A spiral up a cylinder produces a **helix.**

E. To find a vector valued function given an equation in rectangular form, you must find a set of parametric equations for the graph and then rewrite in vector-valued function form.

F. Many of the techniques like adding, subtracting, and scalar multiplication used for real-valued functions can be applied to vector valued functions as well.

G. Definition of the Limit of Vector-Valued Functions:

1. If **r** is a vector-valued function such that **r**(t) = f(t)**i** + g(t)**j**, then

$$\lim_{t\to a}\mathbf{r}(t) = \left[\lim_{t\to a} f(t)\right]\mathbf{i} + \left[\lim_{t\to a} g(t)\right]\mathbf{j}$$

Provided f and g have limits as t approaches a

2. 2. If **r** is a vector-valued function such that **r**(t) = f(t)**i** + g(t)**j** + h(t)**k** then

$$\lim_{t\to a}\mathbf{r}(t) = \left[\lim_{t\to a} f(t)\right]\mathbf{i} + \left[\lim_{t\to a} g(t)\right]\mathbf{j} + \left[\lim_{t\to a} h(t)\right]\mathbf{k}$$

Provided f and g have limits as t approaches a

H. Definition of Continuity of a Vector Valued Function

A vector-valued function r is **continuous at the point** given by t = a if the limit of r(t) exists as t approaches a and $\lim_{t\to a}\mathbf{r}(t) = \mathbf{r}(a)$.

A vector-valued function **r** is **continuous on an interval** I if it is continuous at every point in the interval.