10.7: Cylindrical and Spherical Coordinates

A. The Cylindrical Coordinate System

In a cylindrical coordinate system, a point P in space is represented by an ordered triple (r,θ,z).

1. (r,θ) is a polar representation of the projection of P in the xy-plane.
2. z is the directed distance from (r,θ) to P.

B. Change between rectangular to cylindrical coordinates

1. **Cylindrical to rectangular:**

$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$z = z$$

2. **Rectangular to cylindrical:**

$$r^2 = x^2 + y^2$$

$$\tan\theta = \frac{y}{x}$$

$$z = z$$

The point (0,0,0) is called the pole. Like polar coordinates, cylindrical coordinates are not unique.

C. The Spherical Coordinate System

In a spherical coordinate system, a point P in space is represented by an ordered triple (ρ,θ,ϕ).

1. ρ is the distance between P and the origin, $\rho \geq 0$.
2. θ is the same angle used in cylindrical coordinates for $r \geq 0$.
3. φ is the angle between the positive z-axis and the line segment $\overrightarrow{PQ}$, $0 \leq \varphi \leq \pi$.

Note that the first and third coordinates, ρ and φ, are nonnegative. P is the lowercase Greek letter rho, and φ is the lowercase Greek letter phi.

D. Change between rectangular to spherical coordinates

1. **Spherical to rectangular:**

$$x = \rho \sin\phi \cos\theta$$
$$y = \rho \sin\phi \sin$$
$$z = \rho \cos\phi$$

2. **Rectangular to Spherical:**

$$\rho^2 = x^2 + y^2 + z^2$$
$$\tan\theta = \frac{y}{x}$$
$$\phi = \arccos\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$$

E. Change coordinates between cylindrical and spherical systems

Spherical to cylindrical (r ≥ 0):

$$r^2 = \rho^2 \sin^2\phi$$
$$\theta = \theta$$
$$z = \rho \cos\phi$$

Cylindrical to spherical (r ≥ 0):

$$\rho = \sqrt{r^2 + z^2}$$
$$\theta = \theta$$
$$\phi = \arccos\left(\frac{z}{\sqrt{r^2 + z^2}}\right)$$