10.6: Surfaces in Space

A. Definition of a Cylinder

Let C be a curve in a plane and L be a line not in a parallel plane. The set of all lines parallel to L and intersecting C is called a **cylinder**. C is called the **generating curve** (or **directrix**) of the cylinder, and the parallel lines are called **rulings**.

The equation of a cylinder whose rulings are parallel to one of the coordinate axes contains only the variable corresponding to the other two axes. Ex) A vertical right circular cylinder would have the equation $x^2 + y^2 = a^2$, since the rulings would be parallel to the z axis.

B. Quadric Surface

The equation of a quadric surface in space is a second-degree equation of the form

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

There are six basic quadric surfaces

1. Ellipsoid
2. Hyperboloid of One Sheet
3. Hyperboloid of Two Sheets
4. Elliptic Cone
5. Elliptic Paraboloid
6. Hyperbolic Paraboloid

See pages 765-766 for standard form for each quadric surface and plane traces.

C. **To classify a quadric surface**, begin by writing the surface in standard form. Then determine several traces if graphing by hand. For quadric surfaces not centered at the origin, you can form the standard form by completing the square.

D. Surfaces of Revolution

If the graph of a radius function r is revolved about one of the coordinate axes, the equation of the resulting surface of revolution has one of the following forms.

1. Revolved about the x-axis: $y^2 + z^2 = [r(x)]^2$
2. Revolved about the y-axis: $x^2 + z^2 = [r(y)]^2$
3. Revolved about the z-axis: $x^2 + y^2 = [r(z)]^2$