10.2: Space Coordinates and Vectors in Space

A. The **three-dimensional coordinate system** is constructed by passing a 
z-axis perpendicular to the x and y axes at the origin. Taken as pairs 
the axes determine three coordinate planes: the **xy-plane, the xz-
plane, and the yz-plane**. These coordinate planes separate three-space 
into 8 octants.

B. A point in space is determined by **ordered triples** \((x, y, z)\) where \(x, y,\) 
and \(z\) are as follows.

\[
\begin{align*}
 x &= \text{directed distance from the yz-plane to } P \\
 y &= \text{directed distance from the xy-plane to } P \\
 z &= \text{directed distance from the xy-plane to } P
\end{align*}
\]

* We will use a right-handed system in class, not a left-handed system.

C. Many of the formulas established for the two-dimensions system can be 
extended to 3 dimensions.

**Distance between to point:** 
\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}
\]

A **sphere** centered at \((x_0,y_0,z_0)\) and radius \(r\) can be found using the 
distance formula. The standard equation of a sphere is:

\[
(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 = r^2
\]

The **midpoint of a line segment** with endpoints \((x_1,y_1,z_1)\) and \((x_2,y_2,z_2)\) 
has coordinates

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)
\]

D. Using unit vectors \(\mathbf{i} = \langle 1,0,0 \rangle\) the standard unit vector notation for \(v\) is 
\(\mathbf{k} = \langle 0,0,1 \rangle\)

\[
v = v_1 \mathbf{i} + v_2 \mathbf{j} + v_3 \mathbf{k}
\]

E. Vectors in Space: See page 729