10.1: Vectors in the Plane

A. Quantities such as force, velocity, and acceleration involve magnitude and direction, which can not be characterized by a single real number. A directed line segment is used to represent such a quantity. The directed line segment $\overrightarrow{PQ}$ has initial point P and terminal point Q. It's length (magnitude) is denoted $\left\|\overrightarrow{PQ}\right\|$. Directed line segments that have the same magnitude and direction are equivalent. The set of all line segments equivalent to $\overrightarrow{PQ}$ is a vector in the plane denoted $\vec{v} = \overrightarrow{PQ}$. In typeset it would be denoted v = $\overrightarrow{PQ}$.

B. Definition of Component Form of a Vector in the Plan

If $\vec{v}$ is a vector in the plane whose initial point is the origin and whose terminal points in (v_1, v_2), then the component for of $\vec{v}$ is given by

$$\vec{v} = \langle v_1, v_2 \rangle$$

The coordinates v_1 and v_2 are called the components of $\vec{v}$. If both the initial point and the terminal point lie at the origin, then $\vec{v}$ is called the zero vector and is denoted 0 = $\langle 0, 0 \rangle$.

C. The magnitude of a vector: If P(p_1,p_2) and Q(q_1,q_2) are the initial points of a directed line segment, the component form of the vector $\vec{v}$ represented by $\overrightarrow{PQ}$ is $\langle v_1, v_2 \rangle = \langle q_1 - p_1, q_2 - p_2 \rangle$

$$\left\|\vec{v}\right\| = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2}$$

$$\left\|\vec{v}\right\| = \sqrt{v_1^2 + v_2^2}$$

D. Definitions of Vector Addition and Scalar Multiplication: See Page 718

E. The sum of two vectors can be represented geometrically by positioning the vectors so that the initial point of one coincides with the terminal of the other. The resultant vector is the diagonal of a parallelogram having the two vectors as its adjacent sides.

F. Theorem 10.1: Properties of Vector Operations: See page 719

G. Theorem 10.2: Length of a Scalar Multiple
Let v be a vector and c be a scalar. Then

$$\left\|c\vec{v}\right\| = |c|\left\|\vec{v}\right\|$$

H. Theorem 10.3: Unit Vector in the Direction of v.
If v is a nonzero vector in the plane, then the vector

$$\mathbf{u} = \frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{\|\mathbf{v}\|}\mathbf{v}$$

I. The unit vectors $\langle 1,0\rangle$ and $\langle 0,1\rangle$ are called standard unit vectors in the plan

and are denoted
$$\mathbf{i} = \langle 1,0\rangle$$
$$\mathbf{j} = \langle 0,1\rangle$$

The vector v = v_1**i** + v_2**j** is called a linear combination of **i** and **j**.

J. If **u** is a unit vector such that θ is an angle from the positive x-axis to **u**, then the terminal point of **u** lies on the unit circle and

$$\mathbf{u} = \langle \cos\theta, \sin\theta\rangle = \cos\theta\mathbf{i} + \sin\theta\mathbf{j}$$

It follows that for any other vector v

$$\mathbf{v} = \|\mathbf{v}\|\langle \cos\theta, \sin\theta\rangle = \|\mathbf{v}\|\cos\theta\mathbf{i} + \|\mathbf{v}\|\sin\theta\mathbf{j}$$