Appendix E: Rotation and the General Second-Degree Equation

A. Theorem A.1 – Rotation of Axes

The general equation of the conic

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

where B $\neq$ 0, can be rewritten as

$$A'(x')^2 + C'(y')^2 + D'x' + E'y' + F' = 0$$

by rotating the coordinate axes through an angle θ, where

$$\cot 2\theta = \frac{A - C}{B}$$

The coefficients of the new equation are obtained by making the substitutions

$$x = x'\cos\theta - y'\sin\theta$$
$$y = x'\sin\theta + y'\cos\theta$$

Proof is in the appendix

We will using the result of the proof to obtain A′, B′, etc…

$$A' = A\cos^2\theta + B\cos\theta\sin\theta + C\sin^2\theta$$
$$C' = A\sin^2\theta - B\cos\theta\sin\theta + C\cos^2\theta$$
$$D' = D\cos\theta + E\sin\theta$$
$$E' = -D\sin\theta + E\cos\theta$$
$$F' = F$$

Examples: 6, 10

B. Theorem A.2 (optional) Rotation Invariants

C. Theorem A.3 – Classification of Conics by the Discriminant

The graph of $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$ is, except in the degenerate cases, determined by its discriminant as follows.

1. Ellipse or Circle $B^2 - 4AC < 0$
2. Parabola $B^2 - 4AC = 0$
3. Hyperbola $B^2 - 4AC > 0$

Examples: 20, 22, 28, 30