Section 9.6: Polar Equations of Conics and Kepler's Laws

A. Theorem 9.16 – Classification of Conics by Eccentricity

Let F be a fixed point (focus) and D be a fixed line (directrix) in the plane. Let P be another point in the plane and let e (eccentricity) be the ratio of the distance between P and F to the distance between P and D. The collection of all points P with a given eccentricity is a conic.

1. The conic is an ellipse if $0 < e < 1$.
2. The conic is a parabola if $e = 1$.
3. The conic is a hyperbola if $e > 1$.

B. Theorem 9.17 – Polar Equations of Conics

The graph of a polar equation of the form

$$r = \frac{ed}{1 \pm e\cos\theta} \quad \text{Or} \quad r = \frac{ed}{1 \pm e\sin\theta}$$

is a conic, where $e > 0$ is the eccentricity and $|d|$ is the distance between the focus at the pole and its corresponding directrix.

C. Directrix Location with the focus at the pole. (p 703)

1. Vertical Directrix

$$r = \frac{ed}{1 + e\cos\theta} \qquad x = d$$

$$r = \frac{ed}{1 - e\cos\theta} \qquad x = -d$$

2. Horizontal: $r = \dfrac{ed}{1+e\sin\theta}$ y = d

Directrix

$$r = \frac{ed}{1-e\sin\theta}$$ y = - d

D. Determining a Conic From Its Equation

1. For an ellipse:

$$b^2 = a^2 - c^2 = a^2 - (ea)^2 = a^2(1-e^2)$$

2. For a hyperbola:

$$b^2 = c^2 - a^2 = (ea)^2 - a^2 = a^2(e^2 - 1)$$

Examples: 2, 34, 36

D. Kepler's Law

Kepler's Laws, named after a German astronomer Johannes Kepler, can be used to describe the orbits of the planets about the sun.

1. Each planet moves in an elliptical orbit with the sun as a focus.
2. The ray from the sun to the planet sweeps out equal areas of the ellipse in equal times.
3. The square of the period is proportional to the cube of the mean distance between the planet and the sun.

Example: 58