Section 9.4: Polar Coordinates and Polar Graphs

A. Polar Coordinates (r, θ)

r = directed distance from O to point P

θ = directed angle, counterclockwise from polar axis.

Examples: 2, 4

B. Theorem 9.10 – Coordinate Conversion

The polar coordinates (r, θ) of a point are related to the rectangular coordinates (x, y) of the point as follows.

Polar to Rectangular	Rectangular to Polar
1. $x = r\cos\theta$	1. $\tan\theta = \frac{y}{x}$
2. $y = r\sin\theta$	2. $r^2 = x^2 + y^2$

Examples: 8, 14, 16, 22, (24), 26, 30, (32), 38

C. Distance formula using polar coordinates:

$$d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1 - \theta_2)}$$

Example: 50

D. Theorem 9.11 – Slope in Polar Form

If f is a differentiable function, then the slope of the tangent line to the graph r = f(θ) at the point (r, θ) is

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{f(\theta)\cos\theta + f'(\theta)\sin\theta}{-f(\theta)\sin\theta + f'(\theta)\cos\theta}$$

Example: 54

E. Theorem 9.12 – Tangent Lines at the Pole

If $f(\alpha) = 0$ and $f'(\alpha) \neq 0$, then the line $\theta = \alpha$ is tangent at the pole to the graph $r = f(\theta)$.

Examples: 68, (74)

F. Special Polar Graphs (See page 690)

1. Limacon
2. Cardiod
3. Rose
4. Lemniscate
5. Spiral