Section 9.3: Parametric Equations and Calculus

A. Theorem 9.7 – Parametric Form of the Derivative

If a smooth curve C is given by the equations x = f(t) and y = g(t), then the slope of C at (x, y) is

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}, \quad dx/dt \neq 0$$

- Horizontal Tangent Lines occur when dy/dt = 0.
- Vertical Tangent Lines occur when dx/dt = 0.

Example: 2

B. Higher Order Derivatives

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{dx/dt}$$

$$\frac{d^3y}{dx^3} = \frac{d}{dx}\left[\frac{d^2y}{dx^2}\right] = \frac{\frac{d}{dt}\left[\frac{d^2y}{dx^2}\right]}{dx/dt}$$

Examples: 10, 14, 18, 26, (30)

C. Theorem 9.8 – Arc Length in Parametric Form

Given x = f(t) and y = g(t) does not cross itself on an interval [a,b], the arc length is given by

$$s = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2}\, dt$$

Examples: 36, 42

D. Area of a Surface of Revolution

Given x = f(t) and y = g(t) does not cross itself on an interval [a,b], the area of the surface of revolution formed by revolving C about the coordinate axes is given by

$$S = 2\pi \int_a^b g(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$ (Revolution about the x-axis)

$$S = 2\pi \int_a^b f(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$ (Revolution about the y-axis)

Examples: 50