Section 9.2: Plane Curves and Parametric Equations

A. Definition of a Plane Curve

If f and g are continuous functions of t on an interval I, then the equations

$$x = f(t) \quad \text{and} \quad y = g(t)$$

are called **parametric equations** and t is called the parameter. Taken together, the parametric equations, and the graph are called a **plane curve**, denoted by C.

By plotting points in order of increasing values of t, the curve is traced out in a specific direction. This is called the **orientation of the curve.**

Examples: 1

B. Rewriting a parametric equation as a rectangular equation is called eliminating the parameter.

1. Solve for t in one of the equations.
2. Substitute into the second equation.
3. Simply.

- The Domain may need to be adjusted based upon the original parametric equation
- Trigonometry might need to be used.

Examples: 4, 10, 16, 24

C. Finding parametric equations from rectangular equations.

1. Parametric equations are not unique.
2. Usually let x = t and then replace x with t in the rectangular equation.

- For a line through (x_1,y_1) and (x_2,y_2)

  $x = x_1 + t(x_2 - x_1)$ and $y = y_1 + t(y_2 – y_1)$
- For a circle: $x = h + r\cos\theta$ and $y = k + r\sin\theta$

Examples: 44, 46

D. Definition of a Smooth Curve

A curve represented by $x = f(t)$ and $y = g(t)$ is called smooth if f' and g' are continuous on I and not simultaneously 0. The curve is called piecewise smooth if it is smooth on each subinterval or some partition of I.

E. Cycloid: The path down which a particle will slide from point A to point B in the shortest time is an inverted cycloid passing through points A and B. The amazing part is that a particle starting at rest at any other point of the cycloid between A and B will take exactly the same time to reach B.

Equation of a Cycloid

$$x = a(\theta - \sin\theta)$$

$$y = a(1 - \cos\theta)$$

Example: 56