

Section 9.1: Conics and Calculus

A. A circle is the set of points (x, y) which are a fixed distance r , the radius, away from a fixed point (h, k) , the center.

$$(x - h)^2 + (y - k)^2 = r^2 \quad (\text{See Appendix A if needed})$$

B. Theorem 9.1 – Standard Equation of a Parabola

A parabola is set of all points (x, y) that are equidistant from a fixed line called the directrix and a fixed point called the focus.

The standard form of the equation of a parabola with vertex (h, k) and directrix $y = k - p$ is

$$(x - h)^2 = 4p(y - k) \quad (\text{Vertical Axis})$$

For directrix $x = h - p$, the equation is

$$(y - k)^2 = 4p(x - h) \quad (\text{Horizontal Axis})$$

The focus lies on the axis p units (directed distance) from the vertex. The coordinates of the focus are as follows:

$$(h, k + p) \quad (\text{Vertical Axis})$$

$$(h + p, k) \quad (\text{Horizontal Axis})$$

C. Theorem 9.2 – Reflective Property of a Parabola

Let P be a point on a parabola. The tangent line to the parabola at the point P makes equal angles with the following two lines.

1. The line passing through P and the focus.

2. The line passing through P parallel to the axis of the parabola.

Examples: 10, 12, 14, 22

D. Theorem 9.3 – Standard Equation of Ellipse

An ellipse is the set of all points (x, y) the sum whose distances from two fixed points called the foci is constant.

The standard form of the equation of an ellipse with center (h,k) and major and minor axis of lengths 2a and 2b, where $a > b$, is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad (\text{Major axis is horizontal})$$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \quad (\text{Major axis is vertical})$$

The foci lie on the major axis, c units from the center, with $c^2 = a^2 - b^2$

E. Definition of Eccentricity is given by the ratio

$$e = \frac{c}{a}$$

- $0 < e < 1$
- Measures the ovalness of the ellipse
- As e approaches 0, the ellipse becomes more circular. As e approaches 1, the ellipse becomes more elongated.

Examples: 30, 34, 42

F. Theorem 9.5 – Standard Equation of a Hyperbola

A hyperbola is the set of all points (x, y) for which the absolute value of the differences between the distances from two distinct fixed points called foci is constant.

The standard form of the equation of a hyperbola with the center (h, k) is

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \quad (\text{Transverse axis is horizontal})$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1 \quad (\text{Transverse axis is vertical})$$

The vertices are a units from the center, and the foci are c units from the center. $c^2 = a^2 + b^2$

Horizontal Vertices: $(h \pm a, k)$ Foci: $(h \pm c, k)$

Vertical Vertices: $(h, k \pm a)$ Foci: $(h, k \pm c)$

Horizontal transverse axis equations of asymptotes

$$y = k \pm \frac{b}{a}(x - h)$$

Vertical transverse axis equations of asymptotes

$$y = k \pm \frac{a}{b}(x - h)$$

G. Definition of Eccentricity of a Hyperbola

$$e = \frac{c}{a}$$

- $e > 1$
- The larger e is, the wider the hyperbola opens. The smaller e is, the narrower the hyperbola opens.