Section 8.7: Taylor Polynomial and Approximations

The goal of this section is to show how a polynomial function can be used to approximate other functions. To find a polynomial P that approximates another function f, begin by choosing a number c such that P(c) = f(c). The approximating polynomial is said to be expanded about c or centered at c.

Example: 6

A. Definition of nth Taylor Polynomial and nth Maclaurin Polynomial

If f has n derivatives at c, then the polynomial

$$P_n(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \ldots + \frac{f^n(c)}{n!}(x-c)^n$$

is called the nth Taylor Polynomial for f at c. If c=0, then

$$P_n(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 \ldots + \frac{f^n(0)}{n!}x^n$$

is also called the nth Macclaurin polynomial for f.

Examples: 14, 18

B. Theorem 8.19: Taylor's Theorem

If a function f is differentiable through order n+1 in an interval I containing c, then, for each x in I, there exists z between x and c such that

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \ldots + \frac{f^{n}(c)}{n!}(x-c)^n + R_n(x)$$

where

$$R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-c)^{n+1}$$

- One useful consequence of Taylor's Theorem is that

$$\left|R_n(x)\right| \le \frac{\left|x-c\right|^{n+1}}{(n+1)!} \max\left| f^{(n+1)}(z)\right|$$

Examples: 26, 28