Section 8.6: The Ratio and Root Tests

A. Algebra Review: Factorials

- n! = (n)(n-1)(n-2)…(3)(2)(1)
- 0! = 1

B. Convergence and Divergence Tests (Cont.)

9. Theorem 8.17 – Ratio Test: Let $\sum_{n=1}^{\infty} a_n$ be a series with nonzero terms.
   - $\sum_{n=1}^{\infty} a_n$ converges absolutely if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| < 1$
   - $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| > 1$ or $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \infty$
   - The Ratio Test is inconclusive if $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = 1$

Examples: 14, 16, 30

10. Theorem 8.18 – The Root Test: Let $\sum_{n=1}^{\infty} a_n$ be a series.
   - $\sum_{n=1}^{\infty} a_n$ converges absolutely if $\lim_{n\to\infty}\sqrt[n]{|a_n|} < 1$
   - $\sum_{n=1}^{\infty} a_n$ diverges if $\lim_{n\to\infty}\sqrt[n]{|a_n|} > 1$ or $\lim_{n\to\infty}\sqrt[n]{|a_n|} = \infty$
   - The Root Test is inconclusive if $\lim_{n\to\infty}\sqrt[n]{|a_n|} = 1$

Examples: 36, 40

C. Guidelines for Testing A Series for Convergence or Divergence.

1. Does the nth term approach 0? If not, the series diverges.
2. Is the series one of the special types: geometric, p-series, telescoping, or alternating?
3. Can the Integral Test, the Root Test, or the Ratio Test be applied?
4. Can the series be compared favorably to one of the special types?

C. See page 602 for a summary of Tests for Series.

Other examples: 44, 46, 48, 52