Section 8.5: Alternating Series

A. Convergence and Divergence Tests (Cont.)

8. Theorem 8.14 – Alternating Series Test: Let $a_n > 0$.

$$\sum_{n=1}^{\infty} (-1)^n a_n \quad \text{and} \quad \sum_{n=1}^{\infty} (-1)^{n+1} a_n$$

converge if the following two conditions are met.

a) $\lim_{n \to \infty} a_n = 0$

b) $a_{n+1} \le a_n$, for all n

Examples: 10, 12, 20

Absolute vs Conditional Convergence

- $\sum_{n=1}^{\infty} a_n$ is absolute convergent if $\sum_{n=1}^{\infty} |a_n|$ converges.
- $\sum_{n=1}^{\infty} a_n$ is conditionally convergent if $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} |a_n|$ diverges.

Examples: 42, 44, 46

B. Theorem 8.15: If a convergent alternating series satisfies the condition $a_{n+1} \le a_n$, then the absolute value of the remainder R_N involved in approximating the sum of S by S_N is less than or equal to the first neglected term.

- $\left|S - S_N\right| = \left|R_N\right| \leq a_{N+1}$
- You can approximate an infinite sum by a partial sum from by calculating the following

$$S_n \mp a_{n+1} \leq S \leq S_n \pm a_{n+1}$$

- If a_{n+1} is negative then you would subtract to find the upper bound and add to find the lower bound.

Examples: 30, 36