Section 8.3: The Integral Test and p-Series

A. Convergence and Divergence Tests (Cont.)

4. **Theorem 8.10 - Integral Test:** If f is positive, continuous, and decreasing for $x \geq 1$ and $a_n = f(n)$,

   then $$\sum_{n=1}^{\infty} a_n \quad \text{and} \quad \int_1^{\infty} f(x)dx$$

   either both converge or both diverge.

   Examples: 2, 6, 8

5. **Theorem 8.11 - p-Series Test:** The p-series

   $$\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \ldots$$

   - If $p > 1$, then the series converges
   - If $p \leq 1$, then the series diverges

B. Additional Ideas

- When $p = 1$: $\sum_{n=1}^{\infty} \frac{1}{n}$ is called the harmonic series.
- $\lim_{n \to \infty} \frac{1}{n} = 0$, converges as a sequence, but $\sum_{n=1}^{\infty} \frac{1}{n} \to \infty$ and diverges as a series.

  Examples: 14, 18

C. If $\sum_{n=1}^{\infty} a_n$ converges to S, then the remainder $R_N = S - S_N$ is bounded by $0 \leq R_N \leq \int_N^{\infty} f(x)dx$.

Examples: 38, (46)
Other Examples: 54, 56, 60, 62