Section 8.2: Series and Convergence

A. $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \ldots + a_n + \ldots$ is an infinite series.

B. Definition of Convergent and Divergent Series

For the infinite series $\sum a_n$, the nth partial sum is given by

$$S_n = \sum_{n=1}^{N} a_n = a_1 + a_2 + a_3 + \ldots + a_n$$

If the sequence of partial sums converges to S, then the series converges. The limit S is called the sum of the series.

$$S = a_1 + a_2 + a_3 + \ldots + a_n + \ldots$$

If $\{S_n\}$ diverges, then the series diverges.

Examples: 4, 6

C. Convergence and Divergence Tests

1. Theorem 8.6: Geometric Series Test, $\sum_{n=0}^{\infty} ar^n$ a geometric series.

- If $|r| < 1$, the series converges to $\dfrac{a}{1-r}$
- If $|r| \geq 1$, the series diverges

Example: 10, 24, 30

2. Theorem 8.9: Divergent Test: Consider the series, $\sum_{n=1}^{\infty} a_n$

- If $\lim_{n\to\infty} a_n \neq 0$, then the series diverges
- If $\lim_{n\to\infty} a_n = DNE$, then the series diverges
- If $\lim_{n\to\infty} a_n = 0$, no conclusion can be made. However if the series does converge, then $\lim_{n\to\infty} a_n = 0$.

Examples: 14, 16

3. Telescoping Series are of the form (b1 – b2) + (b2 – b3) + (b3 – b4) + (b4 – b5) + …

- The series will only converge if and only if b_n approaches a finite number as n approaches infinity.

Examples: 36, 54, 56, 74

D. Writing repeating decimals as a geometric series and find the infinite sum.