Section 8.1: Sequences

A. A sequence is a function whose domain is the set of positive integers. However, often the notation is different from typical function notation

1. Notation: Domain: 1 , 2 , 3 , 4, … n
   Range: $a_1, a_2, a_3, a_4, \ldots\ a_n$

2. The number a_n is the nth term.
3. Nonrecursive and Recursive Formulas
4. Graphing a Sequence
5. Factorials

Examples: 2, 12, 18, 24, 68, 70, 72

C. Definition of the Limit of a Sequence

Let L be a real number. The limit of a sequence $\{a_n\}$ is L, written as $\lim_{n\to\infty} a_n = L$

Sequences that have limits converge, whereas sequences that do not have limits diverge.

C. Theorem 8.1: Limit of a Sequence

Let L be a real number. Let f be a function of a real variable such that

$$\lim_{x\to\infty} f(x) = L \text{ and } f(n) = a_n$$

then

$$\lim_{n\to\infty} a_n = L$$

(Note: The converse is Not True. If $\lim_{n\to\infty} a_n = L$ *then* $\lim_{x\to\infty} f(x) = L$ *may or may not be true.)*

Examples: 40, 48, 54, 60

D. Properties of Limits of Sequences (See Text)

E. Theorem 8.3: Squeeze Theorem for Sequences

If $\lim_{n\to\infty} a_n = L = \lim_{n\to\infty} b_n$ and there exists an integer N such that $a_n \le c_n \le b_n$ for all n > N, then $\lim_{n\to\infty} c_n = L$

F. Theorem 8.4: Absolute Value Theorem

For the sequence $\{a_n\}$, if

$$\lim_{n\to\infty} |a_n| = 0 \quad \text{then} \quad \lim_{n\to\infty} a_n = 0$$

G. A sequence is monotonic if its terms are nonincreasing or if its terms are nondecreasing.

H. Definition of a bounded Sequence

1. A sequence is bounded above if there is a real number M such that $a_n \le M$ for all n. The number M is called an upper bound of the sequence.

2. A sequence is bounded below if there is a real number N such that $N \le a_n$ for all n. The number N is called a lower bound of the sequence.

6. A sequence is bounded if it is bounded above or bounded below.

I. Theorem 8.5: If a sequence is bounded and monotonic, then it converges.

Examples: 82, 84