Section 8.10: Taylor and Maclaurin Series

A. Theorem 8.22 – The Form of a Convergent Power Series

If f is represented by a power series $\sum_{n=0}^{\infty} a_n (x-c)^n$ for all x in an open interval I containing c, then $a_n = f^{(n)}(c)/n!$ and

$$f(x) = f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n + \ldots$$

B. Definition of Taylor and Macclaurin Series

If a function f has derivatives of all orders at x = c, then the series

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n = f(c) + f'(c)(x-c) + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n + \ldots$$

is called the Taylor series of f(x) at c. If c = 0, then the series is the Maclaurin series of f.

C. Theorem 8.23 – Convergence of Taylor Series

Let $R_n = \frac{f^{(n+1)}(z)}{n!}(x-c)^{n+1}$

If $\lim_{n\to\infty} R_n = 0$ for all x in the interval I, then the Taylor Series for f converges and equals f(x),

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n$$

D. Guidelines for Finding a Taylor Series

1. Differentiate f(x) several times and evaluate each derivative at c. Try to recognize a pattern in these numbers.
2. Use the sequence developed in the first step to form the Taylor coefficients $a_n = f^{(c)}(c)/n!$ and determine the interval of convergence from the resulting power series.

$$f(c) + f'(c)(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n + \ldots$$

3. Within this interval of convergence, determine whether or not the series converges to f(x)

Examples: (4), 6, 20, 22

E. A list of Power Series for Elementary Functions is very helpful. See the list on page 638.

- You can use the power series for Elementary Functions to find power series for composite functions also.

F. Uses of Power Series

- Approximating the value of a function to an indicated error
- Approximating the value of a definite integral

Examples: 34, 36, 48, 54