Section 7.8: Improper Integrals

A. An integral is called improper if there are:
1. upper and/or lower limits of integration are infinite
2. f(x) has a finite number of infinite discontinuities

B. Definition of Improper Integrals with Infinite Integration Limits.
1. If f is continuous on the interval [a,∞), then

$$\int_a^\infty f(x)dx = \lim_{b\to\infty}\int_a^b f(x)dx$$

2. If f is continuous on the interval (-∞,b], then

$$\int_{-\infty}^b f(x)dx = \lim_{a\to-\infty}\int_a^b f(x)dx$$

3. If f is continuous on the interval (-∞,∞), then

$$\int_{-\infty}^\infty f(x)dx = \int_{-\infty}^c f(x)dx + \int_c^\infty f(x)dx$$

- In the first two cases, the improper integral converges if the limit exists, otherwise the improper integral diverges.
- In the third case, the improper integral on the left diverges if either improper integral on the right diverges.

Examples: 2, 6, 14, 20, 24

C. Definition of Improper Integrals with Infinite Discontinuities

1. If f is continuous on the interval [a,b) and has an infinite discontinuity at b, then

$$\int_a^b f(x)dx = \lim_{c \to b^-} \int_a^c f(x)dx$$

2. If f is continuous on the interval (a,b] and has an infinite discontinuity at a, then

$$\int_a^b f(x)dx = \lim_{c \to a^+} \int_c^b f(x)dx$$

3. If f is continuous on the interval [a,b], except for some c in (a,b) at which f has an infinite discontinuity, then

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

- If the limits exist, the improper integral converges. If the limit DNE, the improper integral diverges.

Examples: 32, 48

C. Theorem 7.5: A Special Type of Improper Integral

$$\int_1^\infty \frac{dx}{x^p} = \begin{cases} \dfrac{1}{p-1}, & p > 1 \\ diverges, & p \le 1 \end{cases}$$

Example: 49