Section 7.7: Indeterminant Forms and L'Hopital's Rule

A. Theorem 7.3: The Extended Mean Value Theorem

If f and g are differentiable on an open interval (a,b) and continuous on [a,b] such that $g'(x) \neq 0$ for any x in (a,b), then there exists a point c on (a,b) such that

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

B. Theorem 7.4: L'Hopital's Rule

Let f and g be functions that are differentiable on an open interval (a,b) containing c, except possibly at c itself. Assume $g'(x) \neq 0$ for all x in (a,b), except possibly at c itself. If the limit of f(x)/g(x) as x approaches c produces an indeterminant form (0/0, $\pm\infty/\pm\infty$) then

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \lim_{x \to c} \frac{f'(x)}{g'(x)}$$

- L'Hopital's Rule can be applied more than once.
- If you have other indeterminant forms such as $0 \bullet \infty, 1^{\infty}, 0^{0}$ convert to an indeterminant form that applies to L'Hopitals Rule.
  - Using "ln" can be helpful in evaluating the limit.
  - If you have two fractions try combining them.

Examples: 6, 8ab, 18, 22

C. Summary

1. Indeterminant Forms: 0/0, ∞/∞, ∞ - ∞, 0*∞, 0^0, 1^∞, and ∞^0
2. Determinant Forms:

$$\infty + \infty \to \infty$$

$$-\infty - \infty \to -\infty$$

$$0^{\infty} \to 0$$

$$0^{-\infty} \to \infty$$

3. Who wins the race to infinity?

$$g(x) = e^{nx} \quad , \text{n>0}$$

$$f(x) = x^{m} \quad , \text{m>0}$$

$$h(x) = (\ln x)^{n} \ , \text{n>0}$$

Examples: 28, 32, 40, 46, 52