Section 7.5: Partial Fractions

A. Decomposition of N(x)/D(x) into Partial Fractions

1. **Divide if improper:** If N(x)/D(x) is an improper fraction (that is if the degree of the numerator is greater than the degree of the denominator), then divide the denominator into the numerator to obtain

$$\frac{N(x)}{D(x)} = \text{(a polynomial)} + \frac{N_1(x)}{D(x)}$$

where the degree of N1(x) is less than the degree of D(x). Then apply steps 2, 3, and 4.

2. **Factor the denominator:** Completely factor the denominator into prime linear or quadratic factors.

3. **Linear Factors:** For each factor of the form $(px+q)^m$, the partial fraction decomposition must include the following sum of m factors.

$$\frac{A_1}{(px+q)} + \frac{A_2}{(px+q)^2} + \ldots + \frac{A_m}{(px+q)^m}$$

4. **Quadratic Factors:** For each factor in the form $(ax^2+bx+c)^n$, the partial fraction decomposition must include the following sum of n factors.

$$\frac{B_1x+C_1}{(ax^2+bx+c)} + \frac{B_2x+C_2}{(ax^2+bx+c)^2} + \ldots + \frac{B_nx+C_n}{(ax^2+bx+c)^n}$$

5. Once you have your partial fractions, you multiply each fraction by the LCD. The result is called the **basic equation.**

Examples: 2, 4, 6

B. Guidelines for Solving the Basic Equation.

Linear Factors

1. Substitute the roots of the distinct factors into the basic equation.
2. For repeated linear factors, use the coefficients determined in guideline 1 to rewrite the basic equation. Then substitute other convenient values of x and solve for the remaining coefficients.

Quadratic Factors

1. *Expand the basic equation*
2. *Collect the terms according to powers of x.*
3. *Equate the coefficients of like powers to obtain a system of linear equations involving A, B, and C.*
4. *Solve the system of linear equations.*

- Note: If there is only 1 quadratic factor, you can pick two values of x and find the remaining variables by solving the result.

Examples: 8, 10, 14, 46

C. A few things to remember

1. It is not necessary to use the partial fraction techniques on all rational functions.
2. If the integrand is not in reduced form, reducing it may eliminate the need for partial fractions.
3. Partial Fractions can be used with some quotients involving transcendental functions.
4. If the degree of the numerator is higher than the degree of the denominator, perform long division if the integrand can not be reduced.