Section 7.4: Trigonometric Substitutions

A. Trigonometric Substitutions (a > 0)

1. For integrals involving $\sqrt{a^2 - u^2}$, let u = a sin θ. Then $\sqrt{a^2 - u^2}$ = a cos θ, where $-\pi/2 \le \theta \le \pi/2$

2. For integrals involving $\sqrt{a^2 + u^2}$, let u = a tan θ. Then $\sqrt{a^2 + u^2}$ = a sec θ, where $-\pi/2 \le \theta \le \pi/2$

3. For integrals involving $\sqrt{u^2 - a^2}$, let u = a sec θ. Then $\sqrt{u^2 - a^2}$ = ± tan θ, where $0 \le \theta < \pi/2$ or $\pi/2 < \theta \le \pi$

B. Theorem 7.2: Special Integration Formulas (a > 0)

$$\int \sqrt{a^2 - u^2}\,du = \frac{1}{2}\left(a^2 \arcsin\frac{u}{a} + u\sqrt{a^2 - u^2} \right) + C$$

$$\int \sqrt{u^2 - a^2}\,du = \frac{1}{2}\left(u\sqrt{u^2 - a^2} - a^2 \ln\left| u + \sqrt{u^2 - a^2} \right| \right) + C \qquad u > a$$

$$\int \sqrt{u^2 + a^2}\,du = \frac{1}{2}\left(u\sqrt{u^2 + a^2} - a^2 \ln\left| u + \sqrt{u^2 + a^2} \right| \right) + C$$

Examples: 6, 10, 14, 34, 42