Section 7.3: Trigonometric Integrals

A. Guidelines for Evaluating Integrals Involving Sine and Cosine. (Follow in the order given.)

1. **If the power of the sine is odd and positive, save one sine factor and convert the remaining factors to cosines.** Expand and integrate.

$$\int (\sin^{2k+1} x \cos^n x)dx = \int (\sin^2 x)^k \cos^n x \sin x \, dx = \int (1-\cos^2 x)^k \cos^n x \sin x \, dx$$

2. **If the power of the cosine is odd and positive, save one cosine factor and convert the remaining factors to sines.** Expand and integrate.

$$\int (\sin^m x \cos^{2k+1} x)dx = \int \sin^m x(\cos^2 x)^k \cos x \, dx = \int \sin^m x(1-\sin^2 x)^k \cos x \, dx$$

3. **If the powers of both the sine and cosine are even and nonnegative, make repeated use of the identities**

$$\sin^2 x = \frac{1-\cos 2x}{2} \qquad \cos^2 x = \frac{1+\cos 2x}{2}$$

to convert the integrand to odd powers of the cosine. Then proceed as in guideline 2.

Examples: 4, 6, 10

B. Wallis's Formulas

1. If n is odd ($n \geq 3$), then

$$\int_0^{\pi/2} (\cos^n x)dx = \left(\frac{2}{3}\right)\left(\frac{4}{5}\right)\left(\frac{6}{7}\right)\cdots\left(\frac{n-1}{n}\right)$$

2. If n is even ($n \geq 2$), then

$$\int_0^{\pi/2} (\cos^n x)dx = \left(\frac{1}{2}\right)\left(\frac{3}{4}\right)\left(\frac{5}{6}\right)\cdots\left(\frac{n-1}{n}\right)\left(\frac{\pi}{2}\right)$$

- These formulas are also valid if cosine is replaced with sine.

Example: 18

C. Guidelines for Evaluating Integrals Involving Secant and Tangent (Follow in the order given.)

1. If the power of the secant is even and positive, save the secant-squared factor and convert the remaining factors to tangents. Expand and integrate.

$$\int \sec^{2k} x \tan^n x \, dx = \int (\sec^2 x)^{k-1} \tan^n x \sec^2 x \, dx = \int (1+\tan^2 x)^{k-1} \tan x \sec^2 x \, dx$$

2. If the power of the tangent is odd and positive, save a secant-tangent factor and convert the remaining factors to secants. Expand and integrate.

$$\int \sec^m x \tan^{2k+1} x \, dx = \int \sec^{m-1} x (\tan^2 x)^k \sec x \tan x \, dx = \int \sec^{m-1} x (\sec^2 x - 1)^k \sec x \tan x \, dx$$

3. If there are no secant factor and the power of the tangent is even and positive, convert a tangent squared factor to a secant squared factor. Expand and integrate.

$$\int \tan^n x \, dx = \int \tan^{n-2} x (\tan^2 x) \, dx = \int \tan^{n-2} x (\sec^2 x - 1) \, dx$$

Examples: 24, 32

D. Integrals Involving Sine-Cosine Products with Different Angles

$$\sin mx \sin nx = \frac{1}{2}\Big(\cos\big[(m-n)x\big] - \cos\big[(m+n)x\big]\Big)$$

$$\sin mx \cos nx = \frac{1}{2}\Big(\cos\big[(m-n)\big]x + \sin\big[(m+n)x\big]\Big)$$

$$\cos mx \cos nx = \frac{1}{2}\Big(\cos\big[(m-n)\big]x + \cos\big[(m+n)x\big]\Big)$$

Example: 48

Other Examples: 56, 64