Section 7.2: Integration By Parts

A. Theorem 7.1: Integration By Parts

If u and v are functions of x and have continuous derivatives, then

$$\int u dv = uv - \int v du$$

B. Guidelines for Integration by Parts

1. Try letting dv be the most complicated part of the integrand. Then u will be the remaining factor(s) of the integrand.
2. Try letting u be the portion of the integrand whose derivative is a function simpler than u. Then dv will be the remaining factor(s) of the integrand.

C. Summary of Common Integrals Using Integration by Parts

1. $\int x^n e^{ax} dx \qquad \int x^n \sin(ax) dx \qquad \int x^n \cos(ax) dx$

   let $u = x^n$ and let $dv = e^{ax}, \quad \sin(ax)dx, \quad \cos(ax)dx$

2. $\int x^n \ln x dx \qquad \int x^n arc\sin(ax) dx \qquad \int x^n arc\cos(ax) dx$

   let $u = \ln x, \quad \arcsin(ax), \quad \arctan(ax)$ and let $dv = x^n dx$

3. $\int e^{ax} \sin(bx) dx \qquad \int e^{ax} \cos(bx) dx$

   let u = sin(bx) or cos(bx) and let $dv = e^{ax} dx$

Examples: 12, 16, 20, 28, 34, 36

D. **Tabular Method** can be useful with problems involving repeated application of integration by parts. Hint: Useful with integrals in the form of:

$$\int x^n e^{ax} dx \qquad \int x^n \sin(ax) dx \qquad \int x^n \cos(ax) dx$$

- Assign u and dv as usual.
- Make 3 columns: Alternative signs starting with (+), u and its derivatives, v′ and its antiderivatives.
- Differentiate until you obtain 0.
- Antidifferentiate the same number of times.
- Add the products of the diagonals.

Example: 64