Section 6.6: Moments, Centers of Mass, and Centroids

A. There are several applications of integration that are related to mass.

- Force = (mass)(acceleration)

B. Moments and Center of Mass: One-Dimensional System

Let the point masses $m_1, m_2, \ldots, m_n$ be located at $x_1, x_2, \ldots, x_n$.

1. The moment about the origin is $M_o = m_1x_1 + m_2x_2 + \ldots + m_nx_n$.
2. The center of mass is $\bar{x} = \frac{M_o}{m}$, where $m = m_1 + m_2 + \ldots + m_n$ is the total mass of the system.

   - If $M_o = 0$, then the system is said to be in equilibrium.

   Examples: 2, 4

C. Moments and Center of Mass: Two Dimensional System

Let the point masses $m_1, m_2, \ldots, m_n$ be located at $(x_1,y1)$, $(x_2,y2)$, $\ldots$, $(xnyn)$.

1. The moment about y-axis is $M_y = m_1x_1 + m_2x_2 + \ldots + m_nx_n$.

2. The moment about the x-axis is $M_x = m_1y_1 + m_2y_2 + \ldots + m_ny_n$.

3. The center of mass $(\overline{x}, \overline{y})$ or center or gravity is

$$\overline{x} = \frac{M_y}{m} \qquad \overline{y} = \frac{M_x}{m}$$

where $m = m_1 + m_2 + \dots + m_n$ is the total mass of the system.

Example: 10

D. Moments and Center of Mass of a Planar Lamina

Let f and g be continuous functions such that $f(x) \geq g(x)$ on [a,b], and consider the planar lamina of uniform density p bounded by the graphs of y = f(x), y = g(x), and $a \leq x \leq b$.

1. The moments about and x and y axis are

$$M_x = p\int_a^b \left[\frac{f(x)+g(x)}{2}\right]\left[f(x)-g(x)\right]dx$$

$$M_y = p\int_a^b x\left[f(x)-g(x)\right]dx$$

2. The center of mass $(\overline{x}, \overline{y})$ is given by $\overline{x} = \frac{M_y}{m}$ and $\overline{y} = \frac{M_x}{m}$, where $m = p\int_a^b \left[f(x)-g(x)\right]dx$ is the mass of the lamina.

E. Sometimes the center of mass is called a Centroid.

Examples: 14, 26