# Section 6.4: Arc Length and Surfaces of Revolution

## A. Definition of Arc Length

Let the function given by y = f(x) represent a smooth curve on the interval [a,b]. The arc length of f between a and b is

$$s = \int_a^b \sqrt{1+\left[f'(x)\right]^2}\,dx$$

Similarly, for a smooth curve given by x = g(y), the arc length of g between c and d is

$$s = \int_c^d \sqrt{1+\left[g'(y)\right]^2}\,dy$$

Examples: 4, 16

**B. Definition of a Surface of Revolution:** If a graph of a continuous function is revolved about a line, the resulting surface is a **surface of revolution.**

**C. Definition of Area of a Surface Area:** Let y = f(x) have a continuous derivative on the interval [a,b]. The area S of the surface of revolution formed by revolving the graph of f **about a horizontal or vertical axis is**

$$S = 2\pi\int_a^b r(x)\sqrt{1+\left[f'(x)\right]^2}\,dx$$

where r(x) is the distance between the graph of f and the axis of revolution. If x = g(y) on the interval [c,d], then the surface area is

$$S = 2\pi\int_c^d r(y)\sqrt{1+\left[g'(y)\right]^2}\,dy$$

Examples: 34, 38