Section 5.1: Differential Equations: Growth and Decay

A. Solving Differential Equations using Separation of Variables.

- Rewrite the equation so that each variable occurs on only one side of the equation
- Using Leibniz notation (dy/dx) is often preferred.

B. In many **applications,** the rate of change of a variable is proportional to the value of y. If y is a function of time t, the proportion can be written as follows.

$$\frac{dy}{dt} = ky$$

- If the rate of change is inversely proportional: $\frac{dy}{dt} = \frac{y}{k}$

The solution of this differential equation is given by

Theorem 5.1: If y is a differentiable function of t such that $y > 0$ and $y' = ky$, for some constant k, then

$$y = Ce^{kt}$$

C is the **initial value** of y, and k is the **proportionality constant.** Exponential growth occurs when $k > 0$, and exponential decay occurs when $k < 0$

C. Continuous Interest: $A = Pe^{rt}$

D. Newton's Law of Cooling (optional) – The rate of change in the temperature of an object is proportional to the difference between the object's temperature and the temperature of the surrounding medium. See page 378

Examples: 2, 4, 12, 16, 28, 37, 44